

## Question 1

(7 MARKS)
Consider the function $f(x)=\sqrt{9-4 x}$.
a) Find $f(x+h)$
b) Evaluate and rationalize the numerator of

$$
\begin{equation*}
\text { the expression } \frac{f(x+h)-f(x)}{h} \tag{2}
\end{equation*}
$$

c) Evaluate the limit,

$$
\begin{equation*}
\operatorname{limit}_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \tag{1}
\end{equation*}
$$

d) What does the answer in (c) represent?
e) Find the equation of the tangent line to the graph of $f(x)$ at the point where $x=0$.

## Question 2

(7 MARKS)
Profit (in kina) for a manufacturing firm made from producing and selling $x$ units of a product on a daily basis is given by the profit function :

$$
P(x)=-0.003 x^{2}+120 x-500000
$$

a) Calculate the profit obtained from producing and selling:
i. 0 units.
ii. 30000 units.
b) Determine the number of units that must be produced and sold daily to maximize profit.
c) What is the maximum profit?
d) The company's current practice is to manufacture and sell 30000 units daily. Would you recommend to the manager to change or maintain this norm? Explain your answer.

Question 3
(7 MARKS)
A church window consisting of a rectangle topped by a semicircle is to have perimeter $p$ and area $A$.

If the radius of the semicircle is $r$ and length of the rectangle is $\ell$, do the following.
(See diagram)

a) Write an expression for $p$ in terms of $r$ and $\ell$.
b) Write an expression for $A$, the area of the church window, in terms of $r$ and $\ell$. (1)
c) Write an expression for $A$, the area of the church window, in terms of $p$ and $r$.
d) Determine the area of the church window if $p=10$ and $r=1$.

Leave your answer in exact form.

## QUESTION 4

(7 MARKS)
An observer at ground level is at a distance $d$ from a building. The angles of elevation to the bottoms of the windows on the second and third floors are $\alpha$ and $\beta$, respectively. If $x$ is the distance from the base of the building to the bottom of the second floor and $h$ the distance between the bottom of the second floor and the bottom of the third floor windows, do the following.
a) Draw a diagram to represent this information.
b) Find $\operatorname{Tan} \alpha$ and $\operatorname{Tan} \beta$.
c) Express $x$ in terms of $d$ and $\alpha$.
d) Find the distance $h$.

## Question 5

The general term of a geometric sequence is given by $a_{n}=(-1)^{n-1} 2\left(\frac{3}{4}\right)^{n-1}, n=1,2,3, \ldots$.
a) Write down the first 4 terms of the sequence.
b) What is the common ratio $r$ ?
c) Write down the geometric series of the sequence in part (a) and find the sum of the first 12 terms to 2 decimal places.
d) Find the infinite sum of the geometric series.

Question 6
(7 MARKS)
Consider the parallelogram $A B C D$ as shown below.

$A D=a, A B=b$ and angle $B \hat{A} D=\theta$ radians. Diagonal $A C$ has length $d$ and bisects angles $B \hat{A} D$ and $B \hat{C} D$.
a) Determine angles $C \hat{A} D$ and $A \hat{C} D$. Your answer must be in radians.
b) Find angle $A \hat{D} C$.
c) Apply the cosine rule to triangle $A C D$ and show that:

$$
\begin{equation*}
d=\sqrt{a^{2}+b^{2}-2 a b \operatorname{Cos} \theta} \tag{4}
\end{equation*}
$$

## QUESTION 7

As part of an endurance race, Nathaniel needs to swim from $X$ to $Y$ across a fairly straight section of a river as shown below. Nathaniel swims at $1.8 m s^{-1}$ in still water.


If the river flows with a consistent current of $0.3 \mathrm{~ms}^{-1}$, find:
a) the distance from $X$ to $Y$ correct to 2 decimal places.
b) the direction in which Nathaniel must swim so as to get to $Y$.
c) the time Nathaniel will need to get from $X$ to $Y$.

